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the lectures pdfs are available at:

https://www.physics.umd.edu/rgroups/amo/orozco/results/2022/Results22.htm

Correlations in Optics and Quantum Optics; A series of lectures about correlations and coherence 1. November 2022

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Lesson 4

Tentative list of topics to cover:

- From statistics and linear algebra to power spectral densities
- Historical perspectives and examples in many areas of physics
- Correlation functions in classical optics (field-field; intensity-intensity; field-intensity) part ii
- Correlation functions in quantum examples
- Correlations and conditional dynamics for control
- Correlations in quantum optics of the field and intensity
- Optical Cavity QED
- From Cavity QED to waveguide QED.

The first order coherence

$$g^{(1)}(\tau) = \frac{\langle E^*(t)E(t+\tau)\rangle}{\langle |E(t)|^2\rangle} \longrightarrow \ < \cos\big(\phi(t) - \phi(t+\tau)\big) >$$

This is the phase correlation between two optical fields and gives the visibility of the interference fringes for the interference between the two fields. If the phase is changing in time, there is a frequency; a linewidth. This correlation may be complex.

Reference that includes pulsed sources:

Zheyu Jeff Ou

"*Quantum Optics for Experimentalists*"

World Scientific, Singapore 2017

$$E(\mathbf{r},t) = E(\tau) = \frac{1}{\sqrt{2\pi}} \int d\omega E(\omega) e^{-i\omega\tau}, \qquad E(\omega) = \frac{1}{\sqrt{2\pi}} \int d\omega E(\tau) e^{i\omega\tau}.$$

$E(\omega)$ is the frequency component of the optical field, whose probability distribution $P(\{E(\omega)\}) = P(E(\omega_1), E(\omega_2), ..., E(\omega_j), ...)$ determines the statistical properties of the field.

4.2.1 *Continuous Waves (CW) and Stationary Processes*

Consider the correlation function of the optical field:

$$\Gamma(t,\tau) \equiv \langle E^*(t)E(t+\tau)\rangle = \int d\{E(\omega)\} P(\{E(\omega)\}) E^*(t) E(t+\tau), \quad (4.15)$$

so, $\Gamma(t,\tau) = \Gamma(\tau)$ is independent of time t

$$\Gamma(\tau) \equiv \langle E^*(t)E(t+\tau)\rangle = \lim_{T\to\infty} \frac{1}{T} \int_T dt E^*(t) E(t+\tau).$$

$$\Gamma(\tau) = \frac{1}{2\pi}\int d\omega_1 d\omega_2 \langle E^*(\omega_1)E(\omega_2)\rangle e^{i(\omega_1-\omega_2)t}e^{-i\omega_2\tau} \tag{4.17}$$

Since $\Gamma(\tau)$ is unrelated to time t for continuous stationary waves, we must have

$$\langle E^*(\omega_1)E(\omega_2)\rangle = 2\pi S(\omega_1)\delta(\omega_1-\omega_2), \tag{4.18}$$

where $S(\omega_1)$ is some function of ω_1. The expression above shows that there is no phase relation between different frequency components of a continuous stationary field since $\langle E^*(\omega_1)E(\omega_2)\rangle = 0$ for $\omega_1 \neq \omega_2$. Hence, Eq. (4.17) changes to

$$\Gamma(\tau) = \int d\omega S(\omega)e^{-i\omega\tau} \tag{4.19}$$

or

reminiscent of WKK theorem

$$S(\omega) = \frac{1}{2\pi}\int d\tau \Gamma(\tau)e^{i\omega\tau}. \tag{4.20}$$

From Eq. (4.19), we obtain $I = \Gamma(0) = \int d\omega S(\omega)$. So, $S(\omega)$ is regarded as the spectral function of the optical field. The statistical relation in Eq. (4.18) is a signature of continuous stationary fields.

4.2.2 Pulsed Waves and Non-Stationary Processes

Instantaneous Intensity:

$$I(t) = E^*(t)E(t).$$

Total Energy of the pulse:

$$I = \int_{-\infty}^{\infty} dt E^*(t)E(t).$$

Then the correlation function:

$$\begin{aligned}
\Gamma(\tau) &= \int dt E^*(t) E(t+\tau) \\
&= \frac{1}{2\pi} \int dt d\omega_1 d\omega_2 E^*(\omega_1) E(\omega_2) e^{i(\omega_1-\omega_2)t} e^{-i\omega_2\tau} \\
&= \int d\omega_1 d\omega_2 E^*(\omega_1) E(\omega_2) \delta(\omega_1-\omega_2) e^{-i\omega_2\tau} \\
&= \int d\omega |E(\omega)|^2 e^{-i\omega\tau} \qquad \text{where we used } \frac{1}{2\pi}\int dt\, e^{i\omega t} = \delta(\omega) \\
&= \int d\omega S(\omega) e^{-i\omega\tau},
\end{aligned}$$

the spectral relation in Eq. (4.20), reminiscent of the WKK theorem, stands for pulsed light field.

4.2.5 Transform-Limited Pulse; Mode-Locked Optical Fields

For an optical field with a continuous spectrum, if the relationship among all frequency components is fixed, that is, $E(\omega) = A_0\mathcal{E}(\omega)$ where $\mathcal{E}(\omega)$ is a definite function but A_0 may be a complex random variable, then the optical field has a definite pulse shape $\mathcal{E}(t)$:

$$E(t) = A_0 \int d\omega \mathcal{E}(\omega) e^{-i\omega t} \equiv A_0 \mathcal{E}(t). \tag{4.35}$$

Its correlation function is

$$\Gamma(t_1, t_2) = \langle E^*(t_1) E(t_2) \rangle = \langle |A_0|^2 \rangle \mathcal{E}^*(t_1) \mathcal{E}(t_2) \tag{4.36}$$

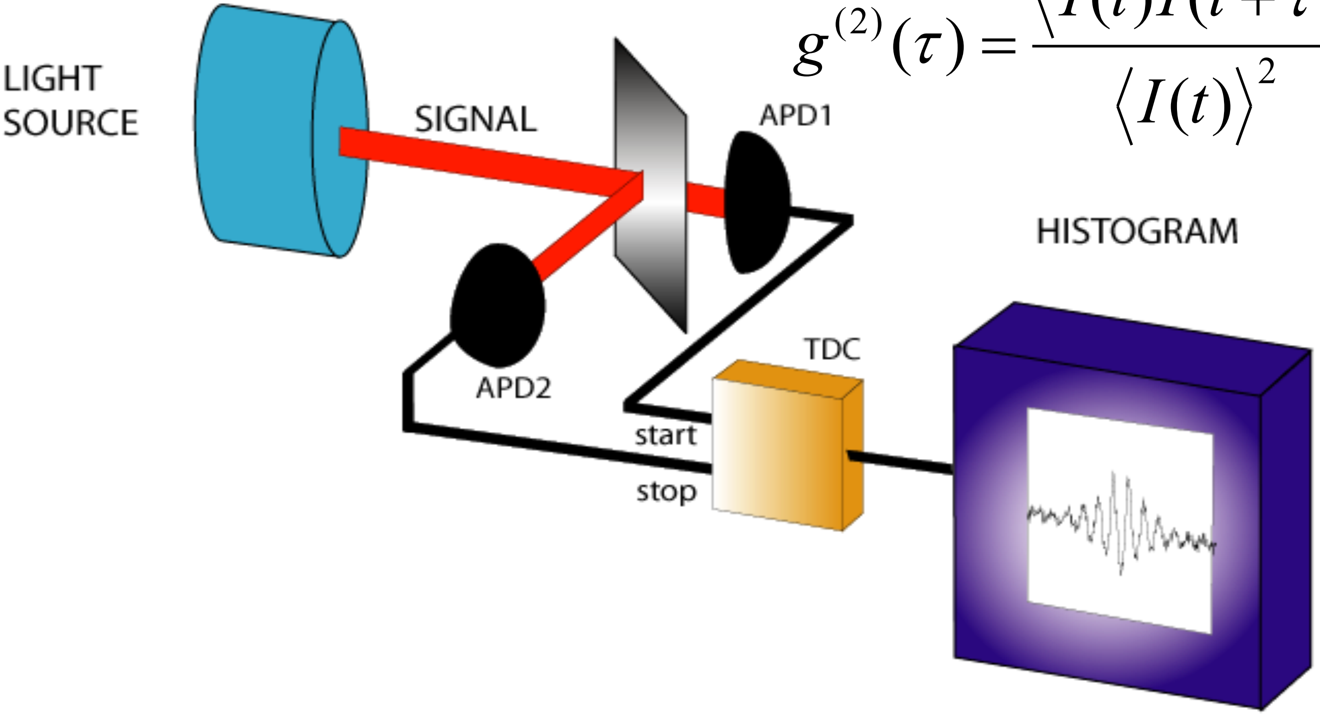
so the normalized correlation function is

$$\gamma(t_1, t_2) = \Gamma(t_1, t_2) / \sqrt{\Gamma(t_1, t_1) \Gamma(t_2, t_2)} = 1. \tag{4.37}$$

The Intensity Intensity Correlation Function

Hanbury Brown and Twiss; Intensity Intensity correlation

$$g^{(2)}(\tau) = \frac{\langle I(t)I(t+\tau)\rangle}{\langle I(t)\rangle^2}$$



R. Hanbury Brown and R.Q. Twiss, Correlation between Photons in Two Coherent Beams of Light, Nature 177, 27 (1956).

Correlations of the intensity at τ=0

$$g^{(2)}(0) = \frac{\langle I(t)^2 \rangle}{\langle I(t) \rangle^2}$$

$$I(t) = I_0 + \delta(t)$$

$$g^{(2)}(0) = \frac{\langle (I_0 + \delta(t))^2 \rangle}{\langle I_0 + \delta(t) \rangle^2}$$

$$(I_0 + \delta(t))^2 = {I_0}^2 + 2I_0\delta(t) + \delta(t)^2$$

$$\langle (I_0 + \delta(t))^2 \rangle = \langle I_0^2 \rangle + 2I_0 \langle \delta(t) \rangle + \langle \delta(t)^2 \rangle = I_0^2 + \langle \delta(t)^2 \rangle$$

$$g^{(2)}(0) = 1 + \frac{\langle \delta(t)^2 \rangle}{I_0^2}$$

It is proportional to the variance, characterizes the statistics

Intensity correlations (bounds)

$$g^{(2)}(0) = 1 + \frac{\langle \delta(t)^2 \rangle}{I_0^2}$$

$$g^{(2)}(0) - 1 \geq 0$$

Cauchy-Schwarz

$$2I(t)I(t+\tau) \leq I^2(t) + I^2(t+\tau)$$

$$|g^{(2)}(\tau) - 1| \leq |g^{(2)}(0) - 1|$$

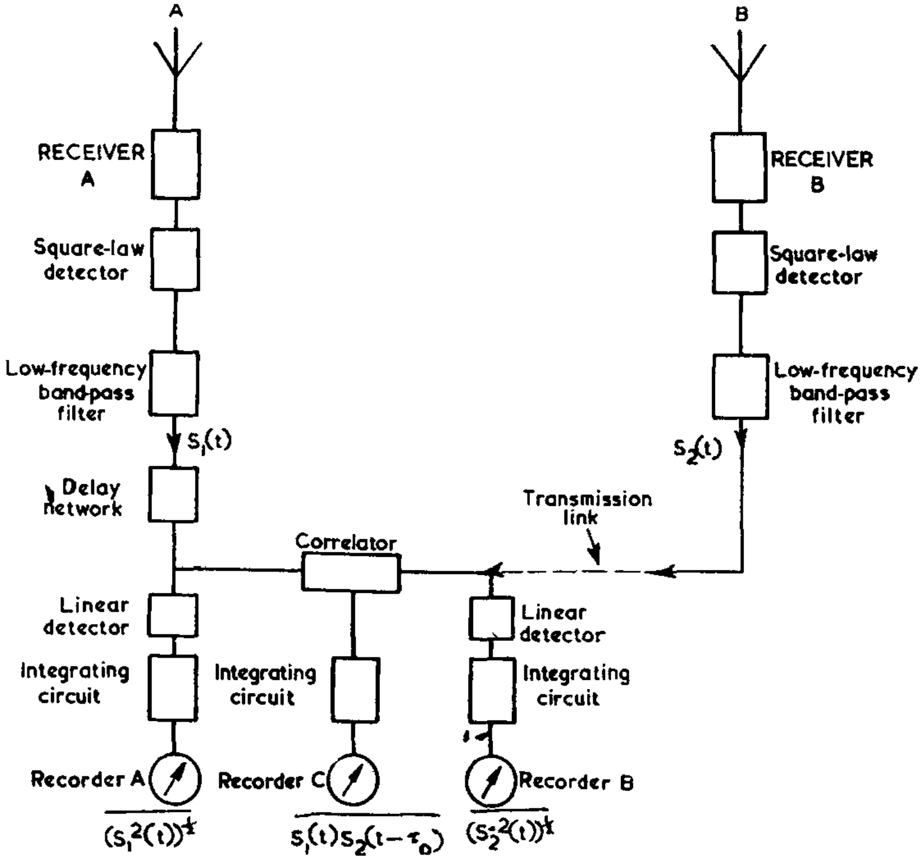
The correlation is maximal at equal times (τ=0) and it can not increase.

How do we measure $g^{(2)}(\tau)$

Original astronomical setup 1952 (analog)

Diode, gives a current proportional to the intensity of the field

Mixer, gives as output the product of the two currents. Then integrate (average).



Block diagram of new type of interferometer.

Digital approach:
The photocurrent is proportional to the intensity I(t)

$$I(t) \to I_i$$

$$I(t+\tau) \to I_j$$

$$\langle I(t)I(t+\tau)\rangle \to \sum_{i=0}^{M}\sum_{n=0}^{N} I_i I_{i+n}$$

- Discretize the time series.
- Apply the algorithm for the correlation the vector.
- Careful with the normalization.

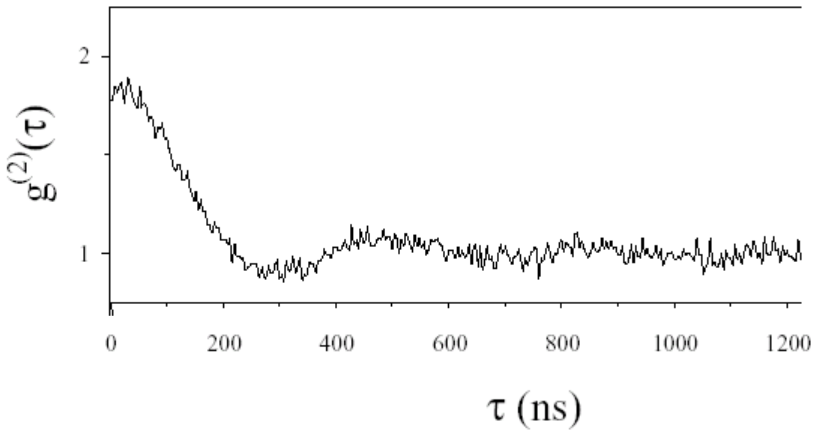
Discretize:

I_i

Displace to multiply (internal product)

I_{i+n}

Add and average:



Another form to measure the correlation with with the waiting time distribution of the photons. The minimum size of the variance of the electromagnetic field.

- Store the time separation between two consecutive pulses (start and stop).
- Histogram the separations
- If the fluctuations are few you get after normalization $g^{(2)}(\tau)$.
- Work at low intensities (low counting rates).

Intensity (photons)

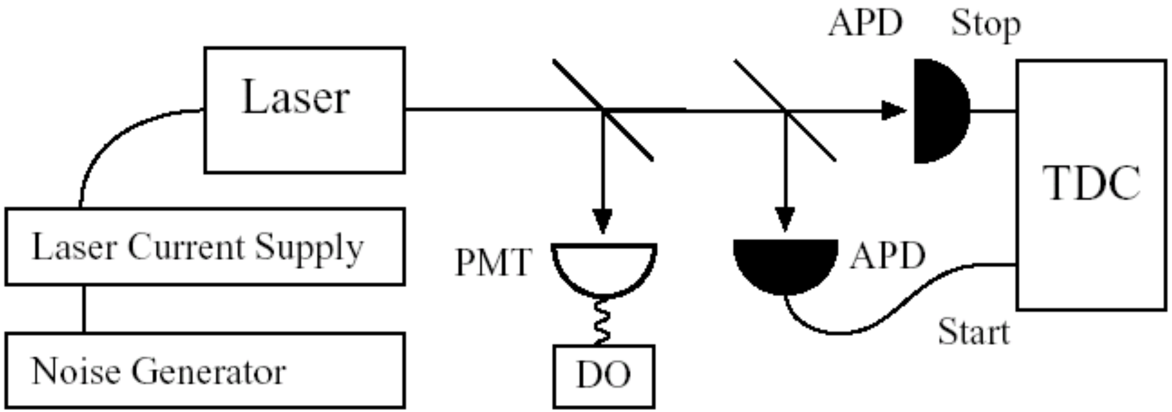
time

Use a time stamp card. Later process the data, then you can calculate all sorts of correlations.

Digitize the full signal (important to identify the nature of the event *e.g.* particle physics).

G. T. Foster, S. L. Mielke, and L. A. Orozco, "Intensity correlations of a noise-driven diode laser," J. Opt. Soc. Am. B, **15,** 2646 (1998).

Simultaneous measurement of $g^{(2)}(\tau)$.



Digital storage oscilloscope (DO) captures the photocurrent from the photomultiplier tube (PMT). The correlation is calculated later

Waiting time distribution with avalanche photodiodes (APD). The time to digital converter (TDC) stores the intervals for later histogramming.

Intensity correlations of a diode laser driven by noise.

$$I(t) = \alpha i(t) \qquad\qquad i(t) = i_0 + i_{noise}(t)$$

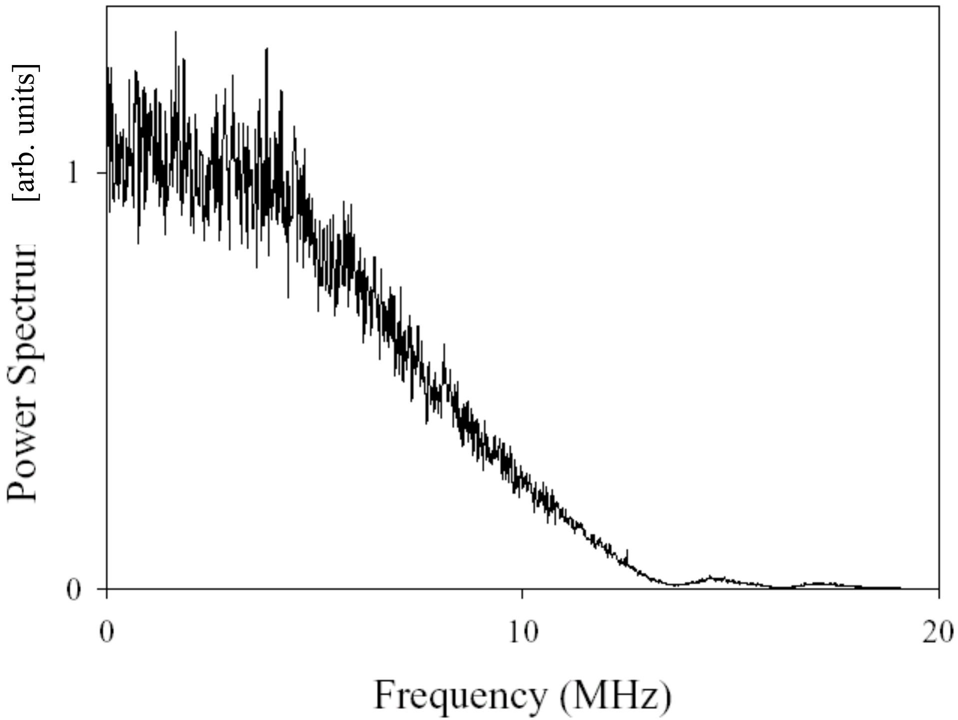
The noise term averages to zero.

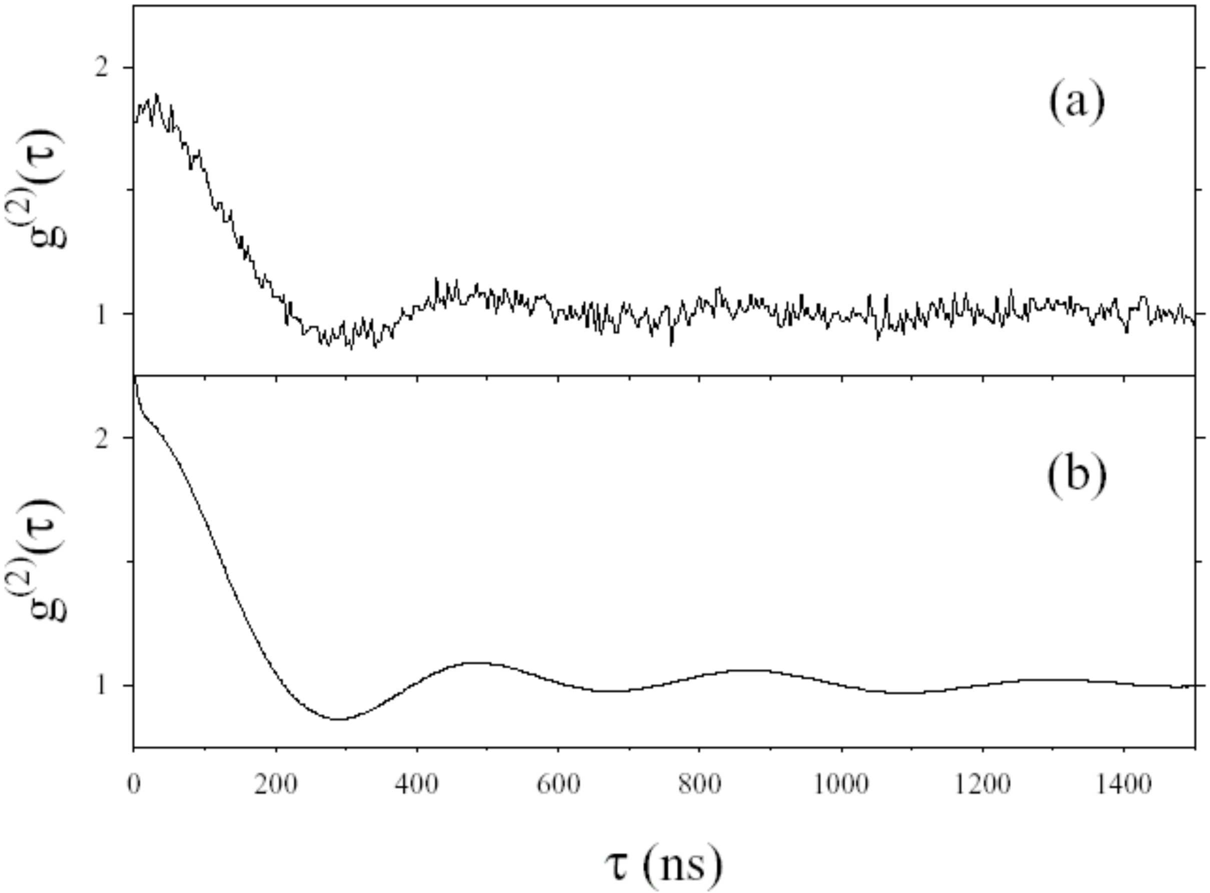
$$g^{(2)}(\tau) = 1 + \frac{\langle i_{noise}(t) i_{noise}(t+\tau) \rangle}{i_0^2}$$

The Wiener-Khintchine-Kolmogorov ttheorem gives the unnormalized correlation function $G^{(2)}(t)$ from the power spectral density of the noise.

$$G(\tau) = \frac{1}{\pi} \int_0^{\infty} F(\omega) e^{-i\omega\tau} dw = \langle i_{noise}(t) i_{noise}(t+\tau) \rangle$$

Power spectrum of the noise source.





Comparison of $g^{(2)}(\tau)$: (a) waiting time distribution of photon counts, (b) time series of photocurrent.

Intensity Field Correlations

Review Article:
H. J. Carmichael, G. T. Foster, L. A. Orozco, J. E. Reiner, and P. R. Rice, "Intensity-Field Correlations of Non-Classical Light". Progress in Optics, Vol. 46, 355-403, Edited by E. Wolf Elsevier, Amsterdam 2004.

- The main object of interest is the optical FIELD. That is what is what is quantized.

- Can we measure the FIELD of a state with an average of ONE PHOTON in it?

Interference of two waves with amplitudes E_1 and ε

$$E_1 e^{i\phi_1} + \varepsilon e^{i\phi_2}$$

$$I = | E_1 e^{i\phi_1} + \varepsilon e^{i\phi_2} |^2$$

$$I = E_1^2 + 2E_1\varepsilon\cos(\phi_1 - \phi_2) + \varepsilon^2$$

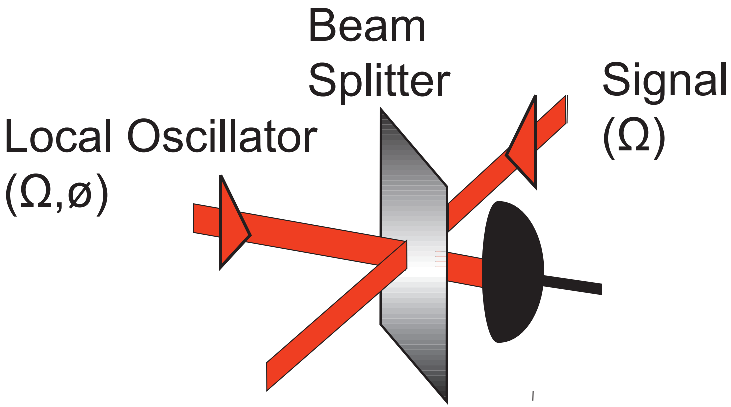
to first order in ε as $E_1 \gg 1$

$$I = I_1 + 2E_1\varepsilon\cos(\phi_1 - \phi_2)$$

homoyne if the phases are equal

heterodyne if the phases are different, may be time dependent.

Homodyne detection



Photocurrent ~ $|L.O.\cos(ø)+S|^2$

Interfere two fields: A local oscillator (LO) and a signal (S). The resulting photocurrent has a term proportional to the amplitude of S and also depends on the cosine of the phase difference ϕ between LO and S.

$$|LO\cos(\phi) + S|^2 = |LO|^2 + 2\,LO\,S\cos(\phi) + |S|^2$$

Review of shot noise :

Shot noise happens whenever the transport of energy is through a finite number of discreet particles. For example, electric charge *e* (Schottky 1918). If the number of particles is small and it follows a Poisson distribution (random independent events), it can be the dominant noise.

- The mean of a Poisson distribution is n
- The variance of a Poisson distribution n
- The signal to noise ratio $n^{1/2}$
- A Poisson distribution with n large approximates a Gaussian.
- The current spectral density (i) of noise is: $(2e|i|)^{1/2}$ with units of [$A/Hz^{1/2}$].
- The power of the noise depends on the detection bandwith and the Resistance R: $P(\nu)=R2e|i|\Delta\nu$.

Review of Coherent States$|\alpha>$

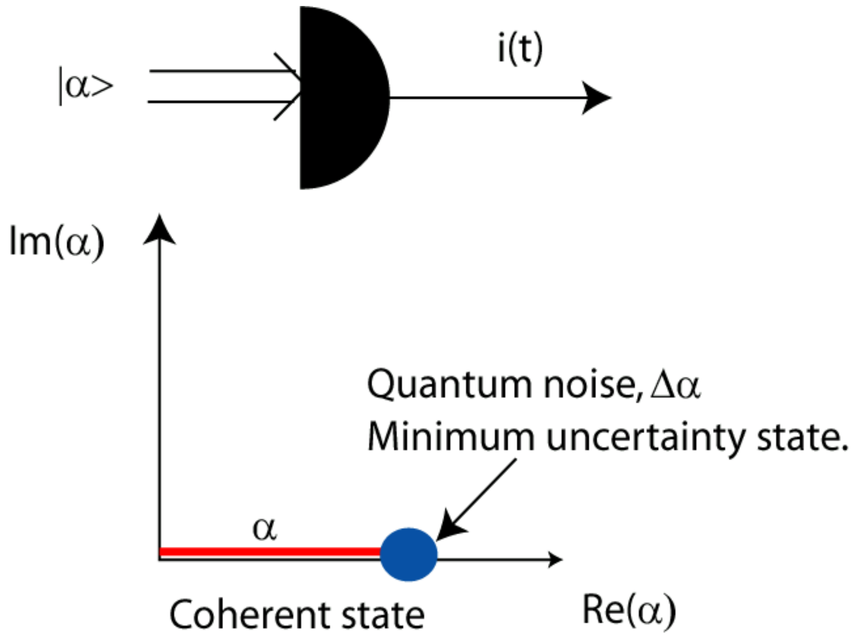
The coherent state $|\alpha>$ is the eigenstate of the annihilation operator:

$$\hat{a}\left|\alpha\right\rangle = \alpha\left|\alpha\right\rangle$$

Its amplitude (complex): α. Its mean squared: $\alpha*\alpha=|\alpha|^2$

Relation to Fock states: $$P(n) = |\langle n|\alpha\rangle|^2 = e^{-\langle n\rangle}\frac{\langle n\rangle^n}{n!}$$

They are states with the minimum uncertainty allowed by quantum mechanics. Equal on both quadratures



Perfect detector $i(t) = |\alpha + \Delta\alpha|^2$

$i(t) = |\alpha|^2 + 2\ \alpha\,\Delta\alpha + |\Delta\alpha|^2$; $\langle \alpha^* \alpha \rangle = n$

DC ~ n   Shot noise ~ $n^{1/2}$ neglect.

Correlation functions tell us something about the fluctuations.

Correlations have classical bounds.

They are conditional measurements.

Can we use them to measure the field associated with a FLUCTUATION of one photon?

Correlation function; Conditional measurement.

Detect a photon: Now follow the evolution of the conditional quantum mechanical state in the system.

The system has to have at least two photons.

Do we have enough signal to noise ratio?

$|LO|^2$ + $2\ LO\ S \cos(\phi)$

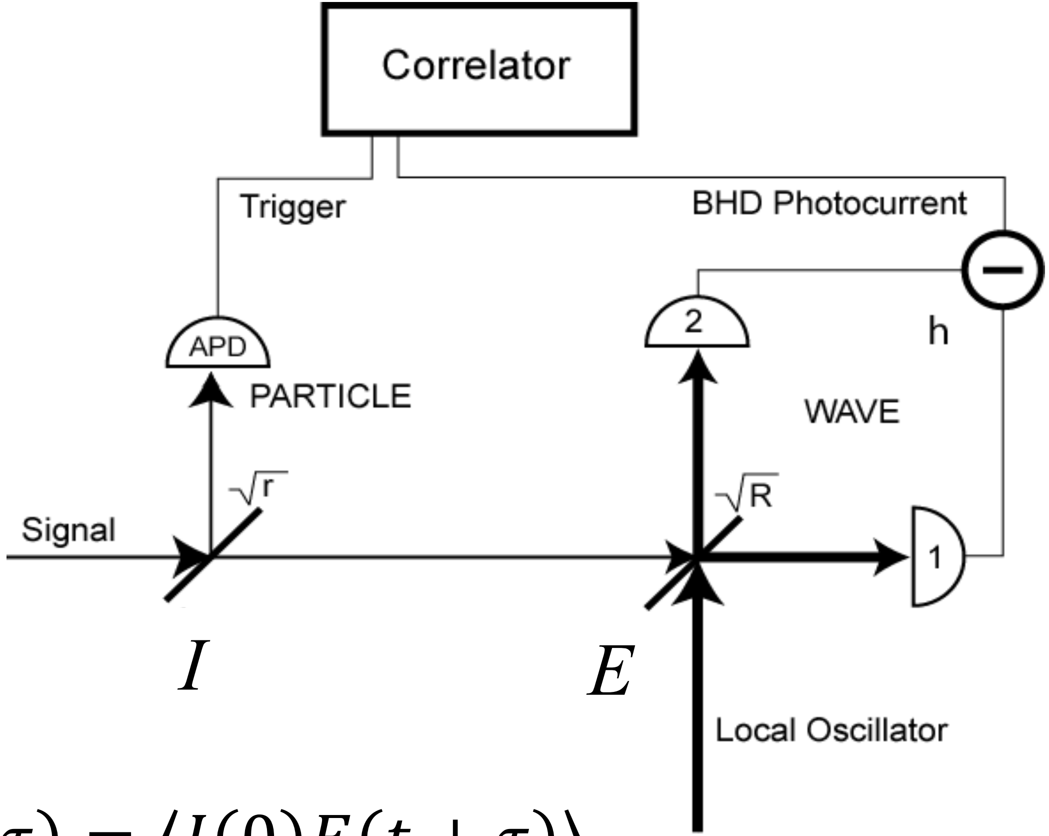
SHOT NOISE SIGNAL

How to correlate fields and intensities?

Detection of the field: Homodyne.

Conditional Measurement: Only measure when we know there is a photon.

The Intensity-Field correlator.



$$H(\tau) = g^{3/2}(\tau) = \langle I(0)E(t+\tau)\rangle$$

Balanced homodyne detector suppresses technical common noise.

The correlator is a digital storage oscilloscope;
Only average the photocurrent if there is a fluctuation.

The average of shot noise is zero.

Condition on a Click

Measure the correlation function of the Intensity and the Field:

$\langle I(t)\, E(t+\tau) \rangle$

Normalized form:

$h_\theta(\tau) = \langle E(\tau) \rangle_c / \langle E \rangle$

From Cauchy Schwartz inequalities:

$$0 \le \bar{h}_0(0) - 1 \le 2$$

$$\left| \bar{h}_0(\tau) - 1 \right| \le \left| \bar{h}_0(0) - 1 \right|$$

Thanks